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ROTATIONAL
PERTURBATIONS IN NUCLEI—
APPLICATION TO WOLFRAM 183

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I. Introduction.

Rotational spectra have been well established as an important feature of the excited states in heavy nuclei. The theory of rotational states and the supporting evidence have been given by BOHR, MOTTELSON, and co-workers (cf. BOHR, 1952; BOHR and MOTTELSON, 1953; BOHR, 1954; BOHR and MOTTELSON, 1954; BOHR, FRÖMAN, and MOTTELSON, 1955; ALAGA, ALDER, BOHR, and MOTTELSON, 1954).

Deviations from the simple rotational spectrum have been found for the most part to be small and to often exhibit the character of rotation-vibration corrections, especially in even-even nuclei (BOHR, 1954). As is well known, such corrections arise as an effect of the centrifugal force on the intrinsic structure of the rotating system. The fact that the observed deviations are usually small implies that the centrifugal force excites only high energy modes of the intrinsic structure. By high energy we mean large compared to the characteristic rotational energy.

An examination of the rotational spectra in Wolfram 183 reveals a deviation from the simple structure, which cannot be accounted for by the rotation-vibration correction. It is proposed that this deviation is caused by the action of the rotation in exciting low energy states of the particle structure. When suitable low energy states exist, the rotational motion is no longer separable and we must treat it as strongly coupled to the degrees of freedom in question. This effect is well known in molecules, where it has been called the Rotational Perturbation (cf., e. g., HERZBERG, 1950, pp. 285-6).

The analogy to molecules, although of great use, must not be carried too far when it comes to detailed discussions. For example, in molecules the heavy particles provide a rather stable field in which the electrons can move; while in the nucleus it is

the nucleons which must provide, in a self-consistent way, the field in which they themselves move. Also, the mass of a nucleon compared to the nucleus is much larger than is the case for electrons in molecules. One may therefore expect that most perturbations will play a more important role in nuclei than they do in molecules.

We can also expect that the rotational perturbations in particular will be less important in even-even nuclei than in odd nuclei, at least near the ground state. The reason for this is the fact that, in even-even nuclei, the first excitation of the particle structure appears to be rather high (~ 1 MeV), probably because of the interaction between pairs of nucleons. Thus, there are no very low energy states which can be coupled to the rotation.

In the following sections, the consequences of the rotational perturbations are developed, using a simplified model at the start in order to keep the essential points foremost. Then, a rather detailed application is made to the accurately measured energy spectrum of Wolfram 183 (cf. MURRAY et al., 1955). From this we obtain good evidence that the interpretation is well founded. A similar analysis of other odd nuclei will be of interest, but this must await the accumulation of more data.

II. Rotational Perturbations.

In order to make clear the origin and generality of the particular rotational perturbation in which we are most interested here, it is useful to consider a simple model. Subsequently, we shall generalize this model in order to make it applicable to the actual case of nuclei. In doing so, the role of other rotational perturbations will become clear.

Therefore, we first restrict ourselves to the system of a single particle coupled to a rigid top by a potential. The Hamiltonian for this system is simply

$$H = \frac{p^2}{2m} + V(\bar{r}) + \sum_{k=1}^3 \frac{\hbar^2}{2\mathfrak{I}_k} (I_k - j_k)^2, \quad (\text{II.1})$$

where \bar{p} , \bar{j} , and \bar{r} are respectively the linear momentum, angular momentum, and position vectors of the particle in the system of

coordinates corresponding to the instantaneous position of the principal axes of the top. The vector I is the total angular momentum of the system; \mathfrak{I}_k are the principal moments of inertia of the top; V is the potential energy between the particle and the top; and m is the reduced mass of the system.

For the analogy with nuclei it is useful to consider first the case where both the top and the potential V have axial symmetry. In particular this means

$$\mathfrak{I}_1 = \mathfrak{I}_2 \equiv \mathfrak{I}. \quad (\text{II.2})$$

The Hamiltonian (1) can then be written in the more convenient form

$$H = H_0 + \frac{\hbar^2}{2\mathfrak{I}_3} (I_3 - j_3)^2 + \frac{\hbar^2}{2\mathfrak{I}} (I^2 - I_3^2 - j_3^2) + RPC, \quad (\text{II.3})$$

with

$$H_0 = \frac{p^2}{2m} + V(\bar{r}) + \frac{\hbar^2}{2\mathfrak{I}} j^2, \quad (\text{II.4})$$

and

$$RPC = -\frac{\hbar^2}{2\mathfrak{I}} (I_+ j_- + I_- j_+), \quad (\text{II.5})$$

where

$$I_{\pm} = I_1 \pm iI_2, \quad j_{\pm} = j_1 \pm ij_2. \quad (\text{II.6})$$

The term defined in equation (5) is an effect of the Coriolis force on the particle and will be given the name "rotation-particle coupling" (*RPC*). When *RPC* is neglected, it is clear because of the axial symmetry, that I_3 and j_3 will be good quantum numbers. These have usually been designated by K and Ω , respectively. The Hamiltonian H_0 can, in principle, be solved for the particle motion with Ω as one of the quantum numbers (for example, see NILSSON, 1955; GOTTFRIED, 1955). Then, the energy spectrum will be given by

$$E = E_{\Omega}^{(0)} + \frac{\hbar^2}{2\mathfrak{I}_3} (K - \Omega)^2 + \frac{\hbar^2}{2\mathfrak{I}} (I(I+1) - K^2 - \Omega^2). \quad (\text{II.7})$$

All available data on rotational spectra in nuclei indicates that only states with $K = \Omega$ occur in the regions of low excitation, i.e. the spectrum has the form

$$E = E_K^{(0)} + \frac{\hbar^2}{2\mathfrak{I}} (I(I+1) - 2K^2). \quad (\text{II.8})$$

This is equivalent to the statement that rotations about the symmetry axis require high energies, or simply that

$$\mathfrak{R}_3 \ll \mathfrak{R}. \quad (\text{II.9})$$

The effects of the rotation-particle coupling can be simply investigated. For this, we first need the non-vanishing matrix elements

$$(IK | I_{\pm} | IK \pm 1) = \sqrt{(I \mp K)(I \pm K + 1)}, \quad (\text{II.10})$$

$$(j\Omega | j_{\mp} | j\Omega \pm 1) = \sqrt{(j \mp \Omega)(j \pm \Omega + 1)}. \quad (\text{II.11})$$

Using these matrix elements we note the general property that the *RPC* preserves the quantum number $(K - \Omega)$ even though it destroys both K and Ω separately. This is an important property, because it means that matrix elements to the lowest lying states, which have $K - \Omega = 0$ in common with the ground state, do not vanish.

Whether or not the *RPC* will be important depends essentially on the spectrum of energies for the Hamiltonian H_0 . If the spacings in this spectrum are large compared to the rotational energies, or, in other words, if the rotation is adiabatic with respect to the particle motion, then the *RPC* is a small perturbation and the simple rotational spectrum (8) is to be expected. This corresponds to a strong coupling of the particle to the rotator. In the opposite limit, when the particle is almost decoupled from the rotator, the quantum numbers Ω and K are not appropriate and the non-spherical part of the coupling energy $V(\bar{r})$ should be treated as the perturbation. We shall be interested in the case of partial decoupling. By this is meant the situation where the great mass of the particle spectrum is high in energy, but where there may be one or a few levels which are low enough so that they cannot be simply treated in perturbation theory. The *RPC*, acting through these levels, partly decouples the particle from the rotator. In the following section, we shall discuss this case in detail.

An important example of partial decoupling is the case where $\Omega = 1/2$ and the rotator has axial symmetry. Because of this symmetry, the states Ω and $-\Omega$ are degenerate and the wave function is a "symmetrical" combination of the two (cf. BOHR, 1952).

Then, for the special case $\Omega = 1/2$, the *RPC* has a diagonal matrix element, and this contributes the so-called "decoupling term" to the rotational spectrum (DAVIDSON and FEENBERG, 1953; BOHR and MOTTELSON, 1953; see also equation (III.1) below). There are many cases of odd nuclei with $\Omega = 1/2$ where this decoupling effect has been met with.

Up to this point, we have been considering the simple model of a particle coupled to an axially symmetric rigid top. We shall now proceed to generalize on this model. Our first step is to relax the restriction as to axial symmetry. This leads to the following four perturbations.

$$H' = H'_1 + H'_2 + H'_3 + V'(\bar{r}) \quad (\text{II.12})$$

$$\equiv \left(\frac{\hbar^2}{8 \mathfrak{I}_1} - \frac{\hbar^2}{8 \mathfrak{I}_2} \right) \left\{ -2(I_+ j_+ + I_- j_-) + (j_+ j_+ + j_- j_-) \right. \\ \left. + (I_+ I_+ + I_- I_-) \right\} + V'(\bar{r}), \quad (\text{II.13})$$

where $V'(\bar{r})$ is the axial asymmetry in the particle potential energy, and the moment of inertia \mathfrak{I} in (7) is now the harmonic mean of \mathfrak{I}_1 and \mathfrak{I}_2 . All of these perturbations are such that they do not preserve the quantum number $(K - \Omega)$. In view of the condition (9), this means that they couple only to high energy states. Therefore, in this case, H' can be treated as a small perturbation. The analogous perturbations have been previously examined by BOHR (1952) for the model of a particle interacting with an incompressible liquid drop, in which case an explicit expression for $V'(\bar{r})$ is also obtained.

It can easily be seen, using equations (10) and (11) and second order perturbation theory, that H'_1 and H'_3 lead to a renormalization of \mathfrak{I} , while $V'(\bar{r})$ and H'_2 lead to a renormalization of $E_K^{(0)}$. The energy H'_3 gives in addition a new type of term which is negative and proportional to $I^2(I+1)^2$.

It is quite clear, both from general ideas of nuclear dynamics and from the data on rotational spectra, that the nuclear rotational motion is not that of a rigid body. In fact, the measured moments of inertia lie between those for irrotational flow and for a rigid body (cf. BOHR and MOTTELSON, 1955). These moments depend

on the shape of the nucleus which, in turn, depends upon the internal nucleon configuration. However, as long as the moments of inertia are essentially constants of the motion, the considerations above will still apply.

Small deviations from the constant values arise from the fact that the rotational motion will distort the nuclear shape somewhat. This distortion is essentially a stretching, which increases the moment of inertia. Therefore, the moment of inertia can be expanded as a function of $I(I + 1)$, and to first order this introduces a term in the energy which is negative and proportional to $I^2(I + 1)^2$. This is the so-called rotation-vibration energy and its magnitude depends on the deformability of the nucleus and on the dependence of the moment of inertia on the deformation. Other effects of the deformability will arise if the rotation causes a deviation from axial symmetry. (The extreme case where the nucleus is not stable with respect to vibrations about axial symmetry, so-called γ -vibrations, has been considered by BOHR and MOTTELSON, 1953, and by WILETS and JEAN, 1955.) Then, as we have already seen, the energies (13) lead to a renormalization of $E_K^{(0)}$ and \mathfrak{J} as well as providing another rotation-vibration-like correction. For regions far removed from closed shells, where the rotational energies are small (< 100 keV), all the evidence is that the rotation-vibration effects are also small. In addition, the evidence suggests that, as one approaches closed shells, these effects will increase to a point where one would have to treat the rotation-vibration coupling in a more exact manner than second order perturbation theory. This would be an analogous situation to the one considered here for the *RPC*. For the nuclei near magic numbers, where one expects a spherical equilibrium shape, the rotation-vibration coupling is so strong that it makes no real sense to speak of rotations. The collective motion then takes the form of oscillations analogous to the surface vibrations of a liquid drop. One has some evidence that this is the case (GOLDHABER and WENESER, 1955).

We must keep in mind the fact that in nuclei the moments of inertia arise as a collective effect of all the particles. Actually, the inertia of the rotational motion is intimately associated with the *RPC*; and in fact the moment of inertia can be traced back to a second order effect of the *RPC* on all the particles (cf. below).

The decoupling effects which we consider here arise in cases where higher order terms are important so that it is necessary to describe more explicitly the corresponding degrees of freedom. Essentially, this is the case when there exist low lying states of excitation of the particle structure, or special degeneracies, as in the case of $\Omega = 1/2$ with axial symmetry. Such low lying excitations, with energies comparable to rotational energies, are usually found in odd-A nuclei and we therefore expect decoupling effects to be especially important in these cases.

Finally, it is emphasized that the *RPC*, which we consider acting between low lying configurations of the rotating nucleus, is of a very general nature (Coriolis force) and its existence is independent of any specific assumptions about the intrinsic nuclear structure.

III. Mixing of Two Rotational Bands.

Let us consider the simplest possible case where the *RPC* will have an important effect. This is the situation when there is a single low-lying configuration which is coupled to the ground state by the *RPC*. In this case, the energy can be simply diagonalized (cf., e.g., HERZBERG, 1950, p. 283) and the various limits considered afterwards. Without the *RPC* there is associated with each of the configurations a rotational band having energies*

$$E_K(I) = E_K^{(0)} + E_K^{(1)} \left\{ I(I+1) + \delta_{K, 1/2} a (-1)^{I+1/2} \left(I + \frac{1}{2} \right) \right\}, \quad (\text{III. 1})$$

where the parameter $E_K^{(1)}$ and the decoupling parameter a depend in some way upon the nucleon configuration, and where $E_K^{(0)}$ is conventionally chosen so that $E_K(K)$ will have the experimental energy.

The rotational spectra corresponding to the close configurations K and $K+1$ will be mixed by the *RPC* (only states with

* For reasons of simplicity, we have disregarded the rotation-vibration term

$$-E_K^{(2)} \left\{ I(I+1) + \delta_{K, 1/2} a (-1)^{I+1/2} \left(I + \frac{1}{2} \right) \right\}^2, \quad 1(a)$$

although there is no difficulty in including it if the accuracy of the data requires this.

the same I can mix) and the resulting energy spectrum will be given by*

$$E(I) = \frac{1}{2} \left\{ E_{K+1}(I) + E_K(I) \right\} \pm \frac{1}{2} \sqrt{\left\{ E_{K+1}(I) - E_K(I) \right\}^2 + 4 A_K^2 (I - K)(I + K + 1)} \quad (III. 2)$$

where

$$A_K = \left| \left(K \left| \frac{\hbar^2}{2\mathfrak{I}} J_- \right| K + 1 \right) \right| \quad (III. 3)$$

$$J_- = \sum_{\text{particles}} j_- \quad (3) a$$

We note that the state $I = K$ is not affected because it has no perturbing partner in the $(K + 1)$ spectrum. It is clear that the parameter A_K has roughly the order of magnitude of a rotational energy, although it may be considerably smaller if the two configurations have a small overlap.

The amplitudes of mixing will be expressed in the following notation. We write the wave functions as

$$\Psi_I^{H,L} = a_I^{H,L} \Psi_{IK} + b_I^{H,L} \Psi_{IK+1}, \quad (III. 4)$$

$$a_I^2 + b_I^2 = 1, \quad (III. 5)$$

where the superscripts H and L denote the higher and lower energy solutions, respectively, and the dependence on other quantum numbers is left implicit. In terms of the ratio

$$R_K(I) = \frac{\left| \left\{ E_{K+1}(I) - E_K(I) \right\} \right|}{2 A_K \sqrt{(I - K)(I + K + 1)}} \quad (III. 6)$$

the mixing amplitudes are

$$a_I^{H,L} = \left\{ 1 + \left[\pm R + \sqrt{1 + R^2} \right]^2 \right\}^{-1/2}, \quad (III. 7 a)$$

$$b_I^H = a_I^L; \quad b_I^L = -a_I^H. \quad (III. 7 b)$$

* In general there is also the possibility for the RCP to connect different states of the same parity both of which have $K = 1/2$. This arises on account of the symmetrization of the wave function discussed before. The considerations are the same as those presented here and the only difference is that we put $K = -1/2$ in the matrix element A_K .

We have conventionally considered the particle state $K + 1$ to be higher than the state K . For the opposite case we have only to interchange the roles of a_I and b_I in (7).

One is interested in the electromagnetic transition probabilities between the mixed states. These will be given in terms of the above mixing amplitudes and the usual γ -ray transition matrix elements. The latter are expressed in general by the formulae (cf. NILSSON, 1955)

$$B(E2) = \frac{5}{16\pi} e^2 \left\{ (I2 I'K' | IK2, K' - K) \right. \\ \left. + b_{E2}^{KK'} (-1)^{I'+K'} (I2 I', -K' | IK2, -K' - K) \right\}^2 (Q^{KK'})^2 \quad (III.8)$$

$$B(M1) = \frac{3}{4\pi} \left(\frac{e\hbar}{2MC} \right)^2 \left\{ (I1 I'K' | IK1, K' - K) \right. \\ \left. + b_{M1}^{KK'} (-1)^{I'+K'} (I1 I', -K' | IK1, -K' - K) \right\}^2 (G^{KK'})^2, \quad (III.9)$$

where Q , G , b_{M1} , b_{E2} depend upon the intrinsic wave functions. For the diagonal transitions (diagonal in K), these have the usual interpretation in terms of electric quadrupole moments and magnetic g -factors.

Thus

$$Q^{KK} = Q_0^K, \quad (III.10)$$

the electric quadrupole moment in the state K ; and

$$G^{KK} = K(g_K - g_R), \quad (III.11)$$

related to the magnetic g -factors in the state K (cf. BOHR and MOTTELSON, 1953, p. 109).

In quadrupole transitions, the off-diagonal matrix elements and the quantities b_{E2} can usually be neglected because they are single-particle effects, while the diagonal matrix elements are essentially the large collective electric quadrupole moments. For magnetic transitions the term containing b_{M1} enters only in the diagonal case $K = K' = 1/2$. The factor which determines the effect of b_{M1} is the ratio (see (14))

$$\begin{aligned}
 f_{I'} &= (-1)^{I+1/2} \frac{(I1 I', -1/2 | I^{1/2} 1, -1)}{(I1 I' 1/2 | I^{1/2} 1 0)} \\
 &= \frac{(-1)^{I+1/2}}{\sqrt{2}} \left\{ \begin{array}{l} 1, \quad I' = I-1 \\ (2I+1), \quad I' = I \\ -1, \quad I' = I+1. \end{array} \right\} \quad (\text{III. 12})
 \end{aligned}$$

The reduced transition probabilities between the mixed states (neglecting the single-particle $E2$ effects) are then

$$\begin{aligned}
 B(E2) &= \frac{5}{16\pi} e^2 \left\{ a_I a_{I'} Q_0^K (I2 I'K | IK 20) \right. \\
 &\quad \left. + b_I b_{I'} Q_0^{K'} (I2 I'K' | IK' 20) \right\}^2 \quad (\text{III. 13})
 \end{aligned}$$

$$\begin{aligned}
 B(M1) &= \frac{3}{4\pi} \left(\frac{e\hbar}{2Mc} \right)^2 \left\{ a_I a_{I'} G^{KK} (I1 I'K | IK 10) \right. \\
 [1 + \delta_{K,1/2} f^{II'} b_{M1}] &+ b_I b_{I'} G^{K'K'} (I1 I'K' | IK' 10) \\
 &+ a_I b_{I'} G^{K'K} (I1 I'K' | IK 11) \\
 &\left. + a_I b_I G^{K'K} (I1 I'K | IK' 1, -1) \right\}^2 \quad (\text{III. 14})
 \end{aligned}$$

with

$$K' = K + 1; \quad G^{K'K'} = -G^{K'K}. \quad (\text{III. 14 a})$$

The transition probabilities are given in the usual way by

$$T(E2) = \frac{4\pi}{75} \frac{1}{\hbar} \left(\frac{\Delta E}{\hbar c} \right)^5 B(E2) \quad (\text{III. 15})$$

$$T(M1) = \frac{16\pi}{9} \frac{1}{\hbar} \left(\frac{\Delta E}{\hbar c} \right)^3 B(M1). \quad (\text{III. 16})$$

We can easily examine the extreme case where the interacting particle states are almost degenerate. If, for example, we take $K \neq 1/2$, $E_K^{(1)} = E_{K+1}^{(1)}$, and assume that the rotational family is degenerate in zero order, the energy spectrum is

$$E(I) = E_K^{(0)} + E_K^{(1)} \left\{ I(I+1) \pm \left(\frac{A_K}{E_K^{(1)}} \right) \sqrt{(I-K)(I+K+1)} \right\}, \quad (\text{III. 17})$$

while the mixing is fifty fifty, i. e.

$$a_I^H = b_I^H = a_I^L = \frac{1}{\sqrt{2}}; \quad b_I^L = -\frac{1}{\sqrt{2}}. \quad (\text{III.18})$$

One sees, therefore, that the decoupling can have a rather large effect on the rotational spectrum, enough in some cases to make it almost unrecognizable as such (cf. also the decoupling effect when $K = 1/2$). It is not expected that this extreme case will occur, except by some accident, but nevertheless it is instructive as an example of the kind of effect which can occur. It is more instructive to examine the opposite extreme where perturbation theory is useful, and this is done in the next section.

IV. Perturbation Limit for *RPC*.

When the energies of the two configurations (K and $K + 1$) are very different, i. e. the difference is large compared to the rotational energies, we expect that the mixing can be treated as a perturbation. The different orders of the perturbation series are simply obtained by an expansion of the equations in Section III. The relevant expansion parameter is the ratio $R_K(I)^{-1}$. It is clear that whether or not perturbation theory is applicable depends to some extent on the total angular momentum I in view of the I dependence of R_K . In general, the perturbation series becomes less useful, the larger I is.

For the energies in the perturbed ground state band, one obtains the formula*

$$E(I) = E_K^{(1)} \left\{ 1 - \frac{A_{K<}}{E_K^{(1)}} \frac{A_{K<}}{\Delta E} \right\} [I(I+1) - K(K+1)] \\ + \frac{A_{K<}^2}{\Delta E} \left\{ \frac{(E_{K'}^{(1)} - E_K^{(1)})}{\Delta E} + \left(\frac{A_{K<}}{\Delta E} \right)^2 \right\} [I(I+1) - K(K+1)]^2 + \dots \quad (\text{IV.1})$$

* The result (1) is for the case where there is only one state ($K \pm 1$) which interacts with the ground state K . Of course, the perturbation result can just as easily be obtained for the more general case where there are an arbitrary number of interacting states. In higher orders, we would then have to include states with $K \pm 2$, etc., and this might change the results in specific cases. Equation (1) is displayed in order to show the kind of effects which may be expected.

where K labels the ground state band, and K' the interacting band ($K' = K \pm 1$). The quantity ΔE is the energy difference, taken as positive, between the interacting configurations. The necessary condition for this expansion is

$$\frac{A_{K'}}{\Delta E} \ll 1. \quad (\text{IV.2})$$

There are two general remarks which can be made on consideration of this result. First, the *RPC* can significantly increase the moment of inertia for the ground state rotational band; and second, it can provide a vibration-rotation-like term which has a positive sign. We shall look first at the former effect.

In second order perturbation theory, one sees that the moment of inertia is increased by the factor (cf. equation (1))

$$\left\{ 1 + \frac{A_{K'}}{E_K^{(1)}} \frac{A_{K'}}{\Delta E} \right\} \quad (\text{IV.3})$$

and, hence, when there are appropriate low-lying configurations, the last few nucleons can have a rather large effect on the moment of inertia, even when perturbation theory is still valid. For example, in a nucleus with $A \approx 200$, each particle can be said to contribute on the average one half of one percent of the moment of inertia, while the last few nucleons might easily give rise to a contribution of ten per cent. Of course, if the contribution is too large, perturbation theory will not be adequate, especially for the higher states in the band. The empirical evidence shows a general tendency for larger moments in the odd nuclei than in neighbouring even nuclei (cf. BOHR and MOTTELSON, 1955; BOHR, FRÖMAN, and MOTTELSON, 1955). This would indicate that the last few nucleons do indeed have low-lying states, and that the *RPC* is operating as described above. In many cases, the difference is so large as to indicate that a perturbation treatment of the *RPC* is not suitable. In the next section we will discuss the case of ${}_{74}\text{W}^{183}$ where this situation exists.

One can recall the remark made earlier that we expect the *RPC* to be rather intimately connected with the moment of inertia. In fact, INGLIS (1954) and BOHR and MOTTELSON (1955) have shown that the total moment of inertia can be derived in time-dependent perturbation theory as an effect of the rotation on the

particle structure; an effect quite equivalent to the *RPC* when one takes into account all of the particles and uses second order perturbation theory.

It is not surprising to see that the second term in (1) is proportional to $I^2(I+1)^2$. The interesting fact is that the coefficient can in principle have a positive sign. For the particular case we have studied here, the positive sign is obtained as long as the moment of inertia of the interacting band (K') is not too large compared to that of the band (K) under consideration. As has been discussed above, this type of term in the energy can arise as a direct effect of the centrifugal force on the moment of inertia (rotation-vibration interaction) or as an effect of the energy H_3 (eq. (II.12)) in second order perturbation theory. In both cases the coefficient is negative. The reason one can have a positive sign from the *RPC* is that the $I^2(I+1)^2$ term first arises in fourth order perturbation theory. (The same is true for the less important perturbation H_1). Since the *RPC* can usually couple to states of lower energy it may be rather more important than the other couplings, even in fourth order. Thus we may expect in some cases to find an anomalously small or even a positive "rotation-vibration"-like correction.

We seem to have an example of both of these effects (renormalization of moment of inertia and positive rotation-vibration correction) in the excited rotational band of ${}_{74}\text{W}^{182}$ which has been assigned as odd parity and $K = 2$ (cf., e.g., ALAGA et al., 1955). At this excitation energy (1.29 MeV for the first state of the band) one expects that the level density is higher than in the ground state region, and then the *RPC* may play a role, as it does in the neighbouring odd isotope. This seems to be the case because the excited band has a higher moment of inertia than the ground state band (15% higher); and also it has a positive rotation-vibration-like correction, while that of the ground state band is, as usual, negative.

In the perturbation limit, the effect of the rotational admixtures on the electromagnetic transition amplitudes can be easily obtained. To first order in $(A_{K<}/\Delta E)$, the mixing amplitudes are*

* This is for the case where the configuration $K+1$ is higher in energy than the configuration K . For the reverse case the roles of a_I and b_I are interchanged (cf. III equation (7)).

$$a_I^L = b_I^H \sim 1; \quad a_I^H = -b_I^L \sim \left(\frac{A_{K<}}{\Delta E} \right) \sqrt{(I-K)(I+K+1)}. \quad (\text{IV.4})$$

Let us first consider transitions between rotational states in the same band. The following results are obtained, using equations (4), (III.13), and (III.14) along with certain properties of the vector addition coefficients. For $E2$ transitions ($I' = I - 2$) and $M1$ transitions ($I' = I - 1$) within a band (K), we have respectively

$$\left. \begin{aligned} B(E2) &= \frac{5}{16\pi} e^2 (I \ 2 \ I' \ K | I \ K \ 2 \ 0)^2 \\ &\left\{ Q_0^K + \left(\frac{A_{K<}}{\Delta E} \right) \sqrt{\frac{2}{3}} Q^{KK'} f_{E2}^{KK'} \right\}^2; \quad K_{<} \neq 1/2 \end{aligned} \right\} \quad (\text{IV.5})$$

$$\left. \begin{aligned} B(M1) &= \frac{3}{4\pi} \left(\frac{e\hbar}{2Mc} \right)^2 (I \ 1 \ I' \ K | I \ K \ 1 \ 0)^2 \\ &\left\{ G^{KK} \left[1 + \delta_{K,1/2} \frac{(-1)^{I+1/2}}{\sqrt{2}} b_{M1} \right] + \left(\frac{A_{K<}}{\Delta E} \right) \sqrt{2} G^{KK'} f_{M1}^{KK'} \right\}^2 \end{aligned} \right\}, \quad (\text{IV.6})$$

where $K' = K \pm 1$ labels the perturbing band. The values of $f^{KK'}$ are given in Table I.

TABLE I.

$f_{M1}^{KK'}$	$f_{E2}^{KK'}$	K'	
$-(K+1)$ $(K-1)$	$-(2K+3)$ $(2K-3)$	$K+1$ $K-1$	K is ground state band
$(K+1)$ $-(K-1)$	$(2K+3)$ $-(2K-3)$	$K+1$ $K-1$	K is excited band

One sees from (5) and (6) that, when the rotational admixtures are small, they do nothing more than "renormalize" what are understood as the electric quadrupole moments and the magnetic g-factors. The I dependence of the transition amplitudes remains the same as they are without the admixtures. The renormalization of Q_0 will be quite unimportant because the correction term, in addition to being small because of $(A_{K<}/\Delta E)$, is made smaller still because the off-diagonal matrix elements $Q^{KK'}$ are

expected to be unimportant compared to the intrinsic quadrupole moments Q_0^K .

For transitions between mixed rotational bands, in the usual case where the quadrupole moments of the two bands are nearly equal, we get simply

$$B(E2) = \frac{5}{16\pi} e^2 (I 2 I' K' | I K 2, K' - K)^2 \left\{ \left(\frac{A_{K_{<}}}{\Delta E} \right) \sqrt{6} Q_0^K + Q_{K'K} \right\}^2 \quad (IV.7)$$

when

$$Q_0^K = Q_0^{K'}; \quad K' = K \pm 1; \quad K_{<} \neq 1/2.$$

This is a result previously derived and used by ALAGA et al. (1955). One can obtain a similar result in the $M1$ amplitudes, but the necessary condition $G^{KK} = G^{K'K'}$ is not likely to be fulfilled. In equation (7) one sees that the off-diagonal particle term can be important because the strong collective term is cut down by the small factor $(A_{K_{<}}/\Delta E)$.

V. Application to ${}_{74}\text{W}^{183}$.

The energy levels in the odd nucleus ${}_{74}\text{W}^{183}$ have been very accurately determined up to an energy of 450 keV from a study of the γ -transitions and internal conversion following the β -decay of ${}_{73}\text{Ta}^{183}$ (MURRAY et al, 1955). The level scheme suggested by MURRAY et al. can be interpreted in terms of two intermixed rotational bands accompanying configurations with $K = 1/2$ and $K = 3/2^*$ (cf. Fig. 1).

A semi-empirical fit of the ground state rotational band, ($K = 1/2$), using the usual formula (III.1) with the correction III. (1a), is successful in a qualitative way, and leads to the parameters $E_{1/2}^{(1)} = 13.027$ keV, $a = 0.1904$, and $E_{1/2}^{(2)} = 0.003182$ keV. The energy of the $K = 1/2$, $I = 9/2$ level (G) using these parameters is only some 8 keV lower than the experimental value.

* This interpretation has also been considered by CHRISTY (1954).

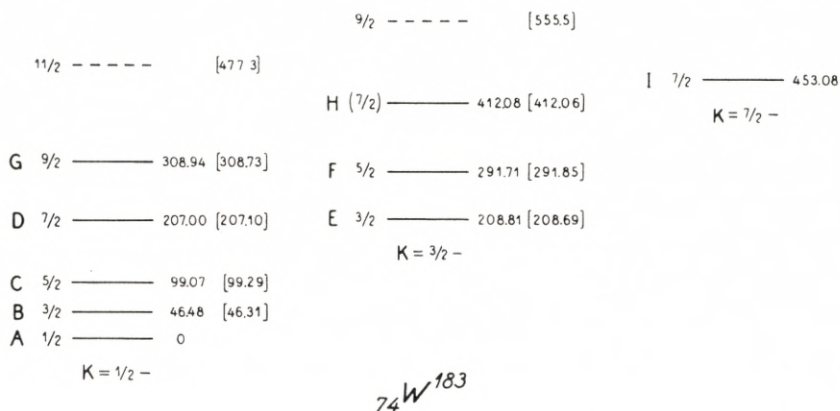


Fig. 1. The level scheme in W^{183} suggested by MURRAY et al. has been drawn so as to display clearly the interpretation in terms of rotational bands. The different bands are horizontally displaced, and the numbers in square brackets are the energies calculated using the parameters (V. 1).

Since it does not fit in with the other rotational sequences we postulate that the level I (spin 7/2) begins a new rotational band with $K = 7/2$. Because the *RPC* cannot directly mix bands with $\Delta K > 1$, the $K = 7/2$ band cannot mix with $K = 1/2$ or $3/2$ bands. Therefore the level I can be disregarded in dealing with the mixing of the other two bands.

The spin for the state *H* has been postulated to be 7/2 rather than 5/2 as suggested by MURRAY et al. This seems to be a reasonable assignment because of the lack of an *E2* transition to the ground state. The spin 7/2, however, is contrary to the multipolarity assignment *M1* for the transition *HE*. This assignment is based on internal conversion evidence (measurement of the *K* conversion coefficient), but appears not to be entirely conclusive. In addition, the theory predicts a 7/2 state with just the properties of the observed level *H*. A conclusive measurement of this spin would of course be very interesting.

This discrepancy, however, is something one would like to understand. In view of (1) the fact that there is a close lying $K = 3/2$ configuration, and (2) that the parameter $E_{1/2}^{(1)}$ (13.027 keV) is 22% less than the value of $E_0^{(1)}$ (16.767 keV) in the neighbouring even-even nucleus $74W^{182}$, it would seem likely that the 8 keV discrepancy is a result of the *RPC*. Then, using the first part of equation (IV.1), we can simply estimate that $A_{1/2} = 28$ keV. This only serves as a first estimate, because the 8 keV discrepancy implies that a fourth order perturbation treatment is not valid. The large effect on the moment of inertia implies the same thing.

The formula (III.2) has been applied to the first five levels above the ground state, and the resulting parameters are

$$\left. \begin{aligned} E_{1/2}^{(1)} &= 15.853 \text{ keV.} & a &= 0.1684 \text{ keV.} \\ E_{3/2}^{(1)} &= 14.050 \text{ keV.} & A_{1/2} &= 21.735 \text{ keV.} \\ E_{3/2}^{(0)} &= 146.74 \text{ keV.} \end{aligned} \right\} \text{(V.1)}$$

The numbers in square brackets in Fig. 1 give the energies as calculated, using these parameters. We note that the discrepancy has been reduced to the order of one fifth of a kilovolt. This remainder might easily be caused by the neglect of other rotation-vibration-like terms in the energy. Just as important, the value of $E_{1/2}^{(1)}$ is now much closer to the analogous quantity in the neighbouring even-even nucleus. Again, the remaining discrepancy might easily be a result of other higher order effects. Finally, it is interesting to note that the matrix element $A_{1/2}$ has the expected order of magnitude of a rotational energy, which we had already seen in the perturbation estimate above.

Making use of the parameters (1) along with equations (III.1), (III.6), and (III.7), a table of mixing amplitudes can be simply constructed.

TABLE II.

I	$3/2$	$5/2$	$7/2$	$9/2$
$a_I^H = -b_I^L$.24	.35	.47	.51
$a_I^L = b_I^H$.97	.94	.88	.86

Now a_I is the amplitude of $K = 1/2$, b_I is the amplitude of $K = 3/2$, and H , L refer to the high and low energy states of the same total angular momentum I . Especially for the higher angular momenta, the mixing is considerable, and one does not expect that perturbation theory would be adequate.

Having determined the mixing amplitudes we can now go on to an examination of the electromagnetic transition probabilities. One has a great wealth of data on relative decay probabilities (cf. MURRAY et al., 1955) and also some data on Coulomb excitation (HUUS et al., 1955; McCLELLAND et al., 1954; MARK et al., 1955; STELSON and MCGOWAN, 1955). Although this data is

quite extensive and accurate it would be very useful to have even more detailed measurements; for example the $E2/M1$ mixtures would be quite interesting and also decisive for the interpretations which follow.

The easiest transitions to discuss are those which are pure $E2$. This is so because here the parameters are roughly known. First, it is not expected that the collective quadrupole moment will vary drastically from one state to another, i.e., we expect $Q_0^{3/2}$ and $Q_0^{1/2}$ to be roughly equal. The fact that the moments of inertia for these two states are closely equal can be considered as evidence for this. Hence, we can take

$$Q_0^{3/2} \approx Q_0^{1/2}. \quad (\text{V.2a})$$

To the same approximation we can also use

$$Q^{3/2^{1/2}} \approx 0$$

because this is a single particle effect.

MURRAY et al. have measured only one intensity ratio for a pair of $E2$ transitions. This is the ratio for the transitions HE and HB in W^{183} (see Fig. 1) which they find to have the value 0.27. A direct calculation, using (2), gives 0.34, which agrees within the accuracy of the measurements. Without the rotational admixture this ratio is too small by a factor ten even for a more favourable choice of $Q^{1/2^{3/2}}$.

Making use of the same assumptions (2) we can construct a table of relative Coulomb excitation probabilities. Thus, fixing $Q_0 \approx 6.5 \cdot 10^{-24} \text{ cm}^2$ to fit the value given by of HUUS et al., (1955), we can calculate the other excitation probabilities. The value of $B_{\text{exc}}(E2)$ for the highest state is roughly one-sixth of the values for the two low states. Without the rotational admixture this would be reduced by more than a factor ten. Recently the excitation AF has indeed been observed (cf. STELSON and MCGOWAN, 1955) and the value of $B_{\text{exc}}(E2)$ determined agrees with the value given in the table, indicating that the estimate (2) is roughly correct. Since the value for Q_0 is consistent with values in the neighbouring nuclei (cf. HUUS et al., 1955) the interpretation seems well confirmed.

TABLE III.

Transition	<i>AB</i>	<i>AC</i>	<i>AE</i>	<i>AF</i>
Energy (keV.)	46.5	99.1	208.8	291.7
$I \rightarrow I^1$	$1/2 \rightarrow 3/2^L$	$1/2 \rightarrow 5/2^L$	$1/2 \rightarrow 3/2^H$	$1/2 \rightarrow 5/2^H$
$B_{exc} \cdot (E2)e^{-2} \cdot 10^{48} \text{ cm}^{-4}$	1.6	2.2	0.099	0.30

The above estimates show that the rotational admixtures play an essential role for certain transitions. In a more quantitative discussion one should consider the quantities $Q_0^{3/2}$, $Q^{1/2 3/2}$, and $b_{E2}^{KK'}$ as parameters to be fitted by the data. The accuracy of the present experiments is not sufficient to determine these parameters although it indicates that the choice (2) is a good one.

There is much more data available with respect to the *M1* transition intensities. MURRAY et al. have measured twelve intensity ratios in W^{183} which can serve as a good test of the theory. Because W^{183} has a ground state spin $I = 1/2$ we see from (III.14) that there are four parameters, leaving eight pieces of data which the theory must fit. The *M1* amplitudes are linear expressions in the four derived parameters α , β , g_3 , g_{13} .

$$\alpha = \left(\frac{\hbar}{MC}\right) \left(1 - \frac{1}{\sqrt{2}} b_{M1}\right) (G^{1/2 1/2}/Q_0^{1/2}), \tag{V.3}$$

$$\beta = \left(\frac{\hbar}{MC}\right) \left(1 + \frac{1}{\sqrt{2}} b_{M1}\right) (G^{1/2 1/2}/Q_0^{1/2}), \tag{V.4}$$

$$g_3 = \left(\frac{\hbar}{MC}\right) (G^{3/2 3/2}/Q_0^{3/2}), \tag{V.5}$$

$$g_{13} = \left(\frac{\hbar}{MC}\right) (G^{1/2 3/2}/Q_0^{1/2}) = -g_{31}. \tag{V.6}$$

Note that the new parameters are defined in units of the ground state quadrupole moment $Q_0^{1/2}$. Since we are dealing with intensity ratios, we are led to a set of twelve quadratic equations in four unknowns. We have proceeded by a trial and error method, and have succeeded in finding values for the parameters which

TABLE IV.

Transition	t_{exp}	t_{theo}	$\% E2$ (theo)	Relative $E2/M2$ phases
BA.....	—	110	.6	(\pm)
CA.....	(28)	28	100	
CB.....	30	20	3	(\pm)
DB.....	(400)	400	100	
DC.....	900	1100	1	(\pm)
EA.....	220	370	20	(\pm)
EB.....	1400	830	10	(\mp)
EC.....	(144)	144	5	(\mp)
FA.....	1330	840	100	
FB.....	—	460	9	(\pm)
FC.....	60	87	70	(\mp)
FD.....	(300)	300	.8	(\mp)
FE.....	47	82	20	(\mp)
GC.....	(1790)	1790	100	
GD.....	65	67	10	(\pm)
HB.....	1860	2500	100	
HC.....	15900	12000	2	(\mp)
HD.....	(1900)	1900	2	(\mp)
HE.....	490	840	100	
HF.....	160	130	50	(\mp)
HG.....	110	280	1	(\mp)

fit the data extremely well considering the uncertainties involved*. The fit is presented in Table IV.

Since transition probabilities are a good test of wave functions, this fit is considered as evidence that the general ideas employed are correct. It would be very interesting to have more measurements, particularly on the $E2/M1$ mixtures. The theory predicts the percentage of $E2$ in the $M1$ radiation, and also the relative phases. Such measurements would be a further very exacting test of the theory, particularly in the cases where there are relatively large admixtures.

* One of the uncertainties in our wave functions has to do with the possibility that the upper ($K = 3/2$) band is perturbed by a still higher band with the same parity and $K = 1/2$ or $5/2$. In fact the high moment of inertia for this band (cf. (1)) indicates that this may be the case.

Caption to Table IV.

We compare here the relative transition probabilities measured by MURRAY et al. to those calculated with the expressions (III. 13) and (III. 14). For convenience, we use natural units ($\hbar = c = 1$; one atomic mass unit = 931 MeV; $10^{13} \text{ cm}^{-1} = 197 \text{ MeV}$; $10^{24} \text{ sec}^{-1} = 658 \text{ MeV}$; $e^2 = 137^{-1}$) and then the parameters which give the fit are (in MeV).

$$\alpha = \mp 0.037 \quad \beta = \mp 0.100 \quad g_3 = \pm 0.042 \quad g_{13} = \pm 0.006.$$

The theoretical quantity tabulated is

$$t = 10^9 T / \left\{ e^2 (Q_0^{1/2})^2 \right\}$$

and this has the dimensions (MeV)⁵ in natural units.

The experimental quantities are relative intensities of γ -ray lines from a given level. Thus we have normalized the intensities in a given group so that one of the experimental values agrees with the corresponding theoretical value for t . This intensity is put in brackets in the table. In the last two columns we have also included the percentage of $E2$ radiation and its phase relative to the $M1$ radiation as predicted by the theory with the above parameters. The $E2/M1$ phases are not unique because a fit of the intensity ratios determines the $M1$ parameters only up to an overall sign relative to the $E2$ parameters. However, once this sign is fixed by a measurement of one of the $E2/M1$ phases, the rest will be determined by the fit. In the table we have given the $E2/M1$ phases to correspond with the signs of the parameters as written above.

The transition FB was not seen by MURRAY et al. because it is masked by very intense radiation of nearly the same energy, originating in level I (compare transitions ID and IE in Fig. 1). Our calculation shows that this transition (FB) is quite comparable in intensity to the others from level F and therefore it should be possible to see it, for example, in Coulomb excitation, where the level I is not excited. Actually the relative intensity FB must be included in an accurate estimate of $B(E2)$ for the level F when this is measured by Coulomb excitation (the value for $B_{\text{exc}}(E2)$ by STELSON and MCGOWAN, is calculated by neglecting FB).

It is probably possible to improve the fit by a more exhaustive analysis using four parameters as above, or by including as parameters the $E2$ matrix elements which have only been roughly estimated here. However, the fit as presented, is probably sufficiently good so that we can say there is some sense in the ideas underlying it.

The percentage $E2$ mixtures in the table are presented only as an indication of the order of magnitude predicted by the theory. The rough estimates as to the $E2$ matrix elements make the values somewhat uncertain.

Conclusion.

We have seen that the *RPC* can have a rather important effect in an odd nucleus as exemplified by ${}_{74}\text{W}^{183}$. It is not expected that this is an atypical case. For example, the systematic effect on the moment of inertia has been noted already (cf. above). In some cases (e.g., ${}_{73}\text{Tl}^{181}$, ${}_{71}\text{Lu}^{175}$, and ${}_{63}\text{Eu}^{153}$), the excited state which might cause a rotational admixture has been seen. More data in such nuclei would be very revealing. From the discussion above it would seem that Coulomb excitation is an excellent tool with which to look for the admixed band, because the mixing greatly enhances the excitation probability for these higher states.

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